Microeconomics

Q&A

Fall 2025

First question: monopoly and mark ups

Monopoly

A **monopolistic market** has one main characteristic:

(1) a single firm that sells to the whole market

No product is a close substitute. This ensures that a **monopolist** is a **price maker**. The demand curve of a monopolist $D(p)$ is simply the market demand curve $X(p)$ for that good. This demand curve is typically downwards sloping

Profit maximization of the monopolist

The monopolist **chooses output** y as to maximize profits while y also affects the price $p(y)$,

$$\max_y p(y)y - c(y).$$

$p(y)$ follows from constrained maximization. The maximization problem of the monopolist is subject to: $D(p) = y$.

Profit maximization of the monopolist

The monopolist **chooses output** y as to maximize profits while y also affects the price $p(y)$,

$$\max_{y} p(y)y - c(y).$$

$p(y)$ follows from constrained maximization. The maximization problem of the monopolist is subject to: $D(p) = y$.

The FOC for profit maximization sets the first derivative to zero,

$$\frac{\partial \pi(y)}{\partial y} = p(y) + \frac{\partial p(y)}{\partial y} y - \frac{\partial c(y)}{\partial y} = 0.$$

Profit maximization of the monopolist

The monopolist **chooses output** y as to maximize profits while y also affects the price $p(y)$,

$$\max_{y}\ p(y)y - c(y).$$

$p(y)$ follows from constrained maximization. The maximization problem of the monopolist is subject to: $D(p) = y$.

The FOC for profit maximization sets the first derivative to zero,

$$\frac{\partial \pi(y)}{\partial y} = p(y) + \frac{\partial p(y)}{\partial y} y - \frac{\partial c(y)}{\partial y} = 0.$$

Which can be written as,

$$\underbrace{p(y) + \frac{\partial p(y)}{\partial y} y}_{MR(y)} = \underbrace{\frac{\partial c(y)}{\partial y}}_{MC(y)}$$

Exercise Ch 14, slide 15

A monopolist faces a demand curve of $D(p) = 11 - p$, has constant marginal costs that are equal to 1, and has fixed costs that are equal to 0.

1. What is the profit-maximizing level of output?

2. What is the accompanying profit?

3. This monopolist can charge a markup. Carefully explain whether a monopolist can always charge a markup.

4. Explain why the markup may be used as a measure of market power.

5. A monopolist's elasticity of demand is 3 and its marginal costs are equal to 10. Calculate the mark-up.

The markup

We have seen that we can write the FOC of the monopolist that $MR = MC$ as:

$$\frac{p(y) - MC(y)}{p(y)} = -\frac{1}{\epsilon(y)}.$$

Define **markup** $= p - MC$.

Whether the monopolist charges a markup depends upon the elasticity of demand. Consider again the three scenarios:

(1) **Demand is completely elastic** with $\epsilon \to -\infty$, then $p = MC$. If demand is completely elastic then the price is given, the monopolist behaves like a perfect competitor and does not ask a markup.

(2) **Demand is elastic** with $-\infty < \epsilon < -1$, then $p > MC$. The monopolist asks a markup, which increases if demand becomes less elastic.

(3) **Demand is inelastic** with $-1 < \epsilon \leq 0$ then $\frac{p-MC}{p} > 1$. This cannot happen since $MC > 0$. Hence, the monopolist will never choose to produce at a point where demand is inelastic.

In other words:

The monopolist has market power in the sense that the amount of output that they can sell responds continuously as a function of the price they charge. Thus, the firm is a price-maker.

In other words:

The monopolist has market power in the sense that the amount of output that they can sell responds continuously as a function of the price they charge. Thus, the firm is a price-maker.

However, the monopolist is still constrained by (i) **consumer's preferences**, which they are summarized by the demand function, and (ii) the technological constraints we have seen in Ch 1.

In other words:

The monopolist has market power in the sense that the amount of output that they can sell responds continuously as a function of the price they charge. Thus, the firm is a price-maker.

However, the monopolist is still constrained by (i) **consumer's preferences**, which they are summarized by the demand function, and (ii) the technological constraints we have seen in Ch 1.

The point is that the monopolist **cannot** consider only (ii) and charge a markup for **any** quantity it produces. Indeed, at different quantity y, the elasticity will be different.

In other words:

The monopolist has market power in the sense that the amount of output that they can sell responds continuously as a function of the price they charge. Thus, the firm is a price-maker.

However, the monopolist is still constrained by (i) **consumer's preferences**, which they are summarized by the demand function, and (ii) the technological constraints we have seen in Ch 1.

The point is that the monopolist **cannot** consider only (ii) and charge a markup for **any** quantity it produces. Indeed, at different quantity y, the elasticity will be different.

Nevertheless, generally, a monopoly operates in a price region such that the elasticity of demand is greater than 1 in absolute value.

In other words:

The monopolist has market power in the sense that the amount of output that they can sell responds continuously as a function of the price they charge. Thus, the firm is a price-maker.

However, the monopolist is still constrained by (i) **consumer's preferences**, which they are summarized by the demand function, and (ii) the technological constraints we have seen in Ch 1.

The point is that the monopolist **cannot** consider only (ii) and charge a markup for **any** quantity it produces. Indeed, at different quantity y, the elasticity will be different.

Nevertheless, generally, a monopoly operates in a price region such that the elasticity of demand is greater than 1 in absolute value.

Therefore, if the monopolist is producing the profit-maximizing quantity and there are no distortions (e.g., taxes), the elasticity will be $|\epsilon| > 1$ and a mark up will be charged.

Elasticity: Profit-maximizing quantity in the $|\epsilon| > 1$ range.

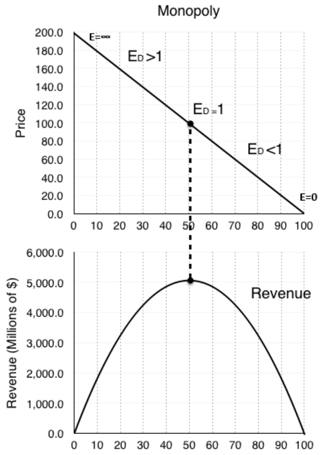


Figure 8.1d

Demand is inelastic ($|\epsilon| > 1$), demand is perfectly elastic ($|\epsilon| = 1$), demand is elastic ($|\epsilon| < 1$).

Special case: Horizontal demand curve (p. 385, MWG)

Consider again the monopolist FOC:

$$\frac{\partial \pi(y)}{\partial y} = p(y) + \underbrace{\frac{\partial p(y)}{\partial y}}_{=0} y - \frac{\partial c(y)}{\partial y} = 0.$$

If $\frac{\partial p(y)}{\partial y} = 0$, the monopolist will price as the price-taking competitive $p = MC$ firm because an increase in price will cause to lose all its sales.

Special case: Horizontal demand curve (p. 385, MWG)

Consider again the monopolist FOC:

$$\frac{\partial \pi(y)}{\partial y} = p(y) + \underbrace{\frac{\partial p(y)}{\partial y}}_{=0} y - \frac{\partial c(y)}{\partial y} = 0.$$

If $\frac{\partial p(y)}{\partial y} = 0$, the monopolist will price as the price-taking competitive $p = MC$ firm because an increase in price will cause to lose all its sales.

Example: Consumers with quasilinear preferences

$$u(q, m) = u(q) + m = \bar{p}q + m$$

where q is the monopolist's good and m is a numeraire. $\bar{p}$ is some value of p.

Special case: Horizontal demand curve (p. 385, MWG)

Consider again the monopolist FOC:

$$\frac{\partial \pi(y)}{\partial y} = p(y) + \underbrace{\frac{\partial p(y)}{\partial y}}_{=0} y - \frac{\partial c(y)}{\partial y} = 0.$$

If $\frac{\partial p(y)}{\partial y} = 0$, the monopolist will price as the price-taking competitive $p = MC$ firm because an increase in price will cause to lose all its sales.

Example: Consumers with quasilinear preferences

$$u(q, m) = u(q) + m = \bar{p}q + m$$

where q is the monopolist's good and m is a numeraire. $\bar{p}$ is some value of p.

Marginal utility of q is constant:

$$MU(q) = \frac{d}{dq}(\bar{p}q) = \bar{p}.$$

A consumer buys any quantity if $p \leq \bar{p}$, and buys zero if $p > \bar{p}$.

Second question: Chapter 10, slide 23. Quasi-linear utility functions and CV/EV.

Chapter 10, slide 23

Consider a consumer with utility function $u = 2\sqrt{x_1} + x_2$ and budget constraint $10 = x_1 + 2x_2$.

1. Calculate the CV for an increase in p_1 from 1 to 2.
2. Calculate the EV for an increase in p_1 from 1 to 2.
3. Why is it that $CV = EV$?
4. Calculate ΔCS for an increase in p_1 from 1 to 2.

Solution

The consumer solves

$$\max_{x_1, x_2} 2\sqrt{x_1} + x_2 \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 = m.$$

The Lagrangian is

$$\mathcal{L} = 2\sqrt{x_1} + x_2 - \lambda(p_1 x_1 + p_2 x_2 - m).$$

First order conditions:

$$\frac{\partial \mathcal{L}}{\partial x_1} : \frac{1}{\sqrt{x_1}} - \lambda p_1 = 0,$$
$$\frac{\partial \mathcal{L}}{\partial x_2} : 1 - \lambda p_2 = 0$$

From the x_2 condition,

$$\lambda = \frac{1}{p_2}.$$

Substituting into the x_1 condition,

$$\frac{1}{\sqrt{x_1}} = \frac{p_1}{p_2} \quad \Rightarrow \quad \sqrt{x_1} = \frac{p_2}{p_1} \quad \Rightarrow \quad x_1(\mathbf{p}, m) = \left(\frac{p_2}{p_1}\right)^2.$$

Plugging x_1 into the budget constraint to find x_2,

$$p_1 \left(\frac{p_2}{p_1}\right)^2 + p_2 x_2 = m$$
$$x_2(\mathbf{p}, m) = \frac{m}{p_2} - \frac{p_2}{p_1}.$$

Solution

So the Marshallian demand is

$$x_1(\mathbf{p}, m) = \left(\frac{p_2}{p_1}\right)^2, \qquad x_2(\mathbf{p}, m) = \frac{m}{p_2} - \frac{p_2}{p_1}.$$

The indirect utility is

$$\begin{aligned} v(\mathbf{p}, , m) &= u\big(x_1(\mathbf{p}, m), x_2(\mathbf{p}, m)\big) \\ &= 2\sqrt{\left(\frac{p_2}{p_1}\right)^2} + \left(\frac{m}{p_2} - \frac{p_2}{p_1}\right) \\ &= \frac{2p_2}{p_1} + \frac{m}{p_2} - \frac{p_2}{p_1} = \frac{m}{p_2} + \frac{p_2}{p_1}. \end{aligned}$$

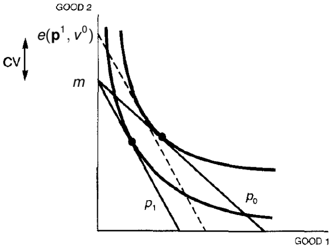
To get the expenditure function, let $e(\mathbf{p}, u)$ be the minimum expenditure needed to reach utility u at prices (p_1, p_2). By definition,

$$u = v\big(\mathbf{p}, e(\mathbf{p}, u)\big) = \frac{e(\mathbf{p}, u)}{p_2} + \frac{p_2}{p_1}.$$

Solve for e:

$$\frac{e(p_1, p_2, u)}{p_2} = u - \frac{p_2}{p_1} \quad \Rightarrow \quad e(p_1, p_2, u) = p_2 u - \frac{p_2^2}{p_1}.$$

Reminder: CV



The **compensating variation** (CV) is defined as:

$$\begin{aligned} CV =& e(\mathbf{p}^1, v^0) - e(\mathbf{p}^0, v^0) \\ =& e(\mathbf{p}^1, v^0) - m. \end{aligned}$$

Old utility v^0 at new price p_1. Recall that a change in p_1 leads to a change in the economic rate of substitution $-p_{x_1}/p_{x_2}$

1. Numerical values for compensating variation (CV)

Take

$$m = 10, \quad p_2 = 2, \quad p_1^0 = 1, \quad p_1^1 = 2.$$

First compute initial utility:

$$v^0 = v(p_1^0, p_2, m) = \frac{m}{p_2} + \frac{p_2}{p_1^0} = \frac{10}{2} + \frac{2}{1} = 5 + 2 = 7.$$

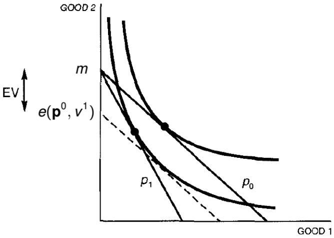
The expenditure needed to reach utility v^0 at the new prices (p_1^1, p_2) is

$$\begin{aligned} e(p_1^1, p_2, v^0) &= p_2 v^0 - \frac{p_2^2}{p_1^1} \\ &= 2 \cdot 7 - \frac{4}{2} = 14 - 2 = 12. \end{aligned}$$

The compensating variation is

$$CV = e(p_1^1, p_2, v^0) - m = 12 - 10 = 2.$$

Reminder: EV



The **equivalent variation** (EV) is defined as:

$$EV = e(\mathbf{p}^1, v^1) - e(\mathbf{p}^0, v^1)$$
$$= m - e(\mathbf{p}^0, v^1).$$

New utility v^1 at old price p^0.

2. Numerical values for equivalent variation (EV)

Utility after the price change at the original income is

$$v^1 = v(p_1^1, p_2, m) = \frac{m}{p_2} + \frac{p_2}{p_1^1} = \frac{10}{2} + \frac{2}{2} = 5 + 1 = 6.$$

The expenditure needed to reach v^1 at the old prices (p_1^0, p_2) is

$$\begin{aligned} e(p_1^0, p_2, v^1) &= p_2 v^1 - \frac{p_2^2}{p_1^0} \\ &= 2 \cdot 6 - \frac{4}{1} = 12 - 4 = 8. \end{aligned}$$

The equivalent variation is

$$EV = m - e(p_1^0, p_2, v^1) = 10 - 8 = 2.$$

Hence $CV = EV = 2$.

3. Why $CV = EV$?

The Marshallian demand for good 1 is

$$x_1(p_1, p_2, m) = \left(\frac{p_2}{p_1}\right)^2,$$

which does not depend on income m. So good 1 has no income effect and the Hicksian and Marshallian demand for good 1 coincide. In this case the three money measures of the price change coincide:

$$CV = EV = \Delta CS.$$

4. Change in consumer surplus

The change in consumer surplus for a price increase from p_1^0 to p_1^1 is

$$\Delta CS = \int_{p_1^0}^{p_1^1} x_1(p_1, p_2, m)\, dp_1 .$$

Here

$$x_1(p_1, p_2, m) = \left(\frac{p_2}{p_1}\right)^2 \quad \text{and} \quad p_2 = 2,$$

so it follows $x_1(p_1, p_2, m) = \dfrac{4}{p_1^2}$ and

$$\Delta CS = \int_1^2 \frac{4}{p_1^2}\, dp_1 = 4 \int_1^2 p_1^{-2}\, dp_1 .$$

Compute the integral:

$$4 \int_1^2 p_1^{-2}\, dp_1 = -4 \frac{1}{p_1}\bigg|_1^2 = -\frac{4}{2} - \left(-\frac{4}{1}\right) = 2.$$

Thus

$$\Delta CS = 2 = CV = EV .$$

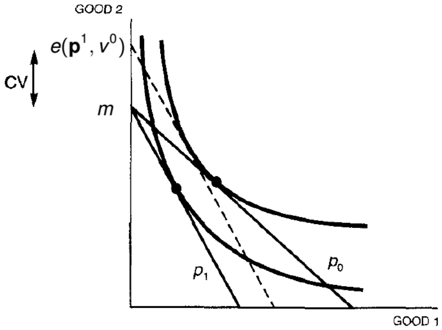
ΔCS is the area left of the Marshallian demand. Here, because utility is quasi linear in x_2, there is no income effect and the Hicksian demand for good 1 equals the Marshallian demand. The area to the left of either demand curve between p_1^0 and p_1^1 is the same. Hence the loss in consumer surplus, the compensating variation and the equivalent variation all have the same numerical value.

Third question: CV represents the amount of income that the consumer will need to receive so that he could achieve his status quo utility (u^0) at the new prices (p').

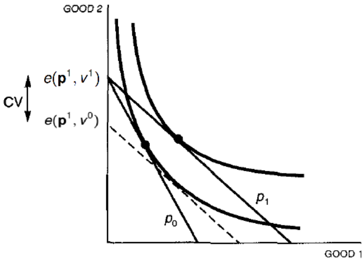
Varian's expression is: $CV = e(p', u') - e(p', u^0)$. In the slides $CV = e(p', u^0) - e(p^0, u^0)$. Why is there a difference between these two expressions?

CV with price increase (A) and price decrease (B)

Panel A: Increase, Slides

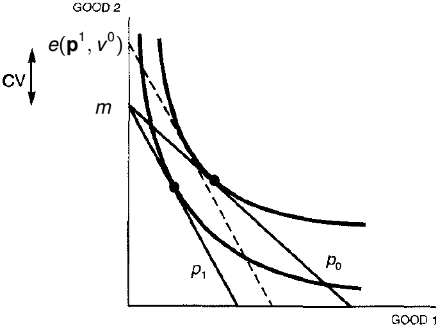


Panel B: Decrease, Varian



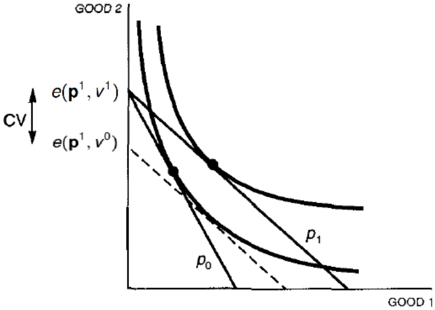
CV with price increase (A) and price decrease (B)

Panel A: Increase, Slides



$$CV = e(\mathbf{p}^1, v^0) - e(\mathbf{p}^0, v^0)$$

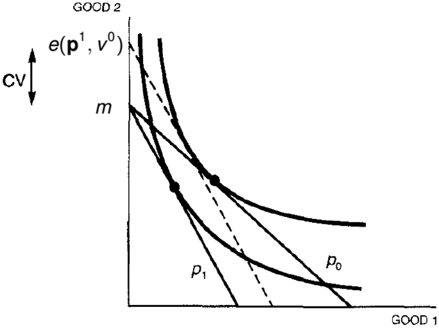
Panel B: Decrease, Varian



$$CV = e(\mathbf{p}^1, v^1) - e(\mathbf{p}^1, v^0)$$

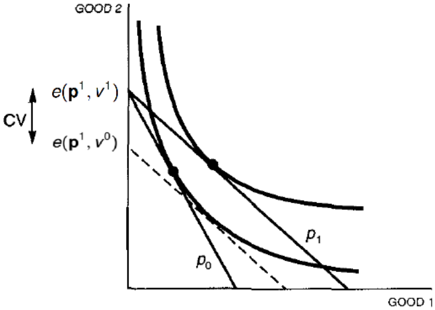
CV with price increase (A) and price decrease (B)

Panel A: Increase, Slides



$$CV = e(\mathbf{p}^1, v^0) - e(\mathbf{p}^0, v^0)$$

Panel B: Decrease, Varian

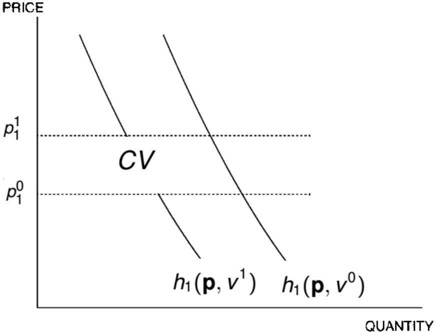


$$CV = e(\mathbf{p}^1, v^1) - e(\mathbf{p}^1, v^0)$$

As long as $e(\mathbf{p}^0, v^0) = e(\mathbf{p}^1, v^1)$, the two expressions are symmetric, accounting for price increase (A) and decrease (B). Turns out this is the case. Recall that the budget constraint is defined as: $m = p_1 x_1 + p_2 x_2$, it follows that $m = e(\mathbf{p}^0, v^0) = e(\mathbf{p}^1, v^1)$.

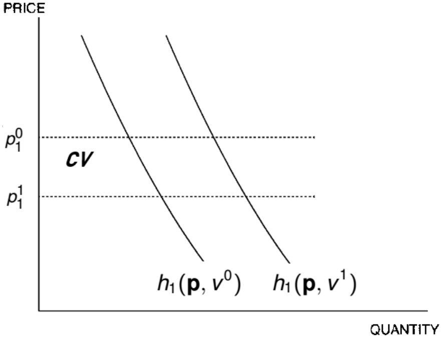
CV

Panel A: Increase, Slides



$$CV = e(\mathbf{p}^1, v^0) - e(\mathbf{p}^0, v^0)$$

Panel B: Decrease, Varian



$$CV = e(\mathbf{p}^0, v^0) - e(\mathbf{p}^1, v^0)$$

Fourth question: Chapter 7, slide 47.

Utility maximization problem (Slide 47, Chapter 2)

Consider the following utility maximization problem:

$$\max_{x_1, x_2} x_1^{\alpha} x_2^{1-\alpha},$$
$$\text{such that } p_1 x_1 + p_2 x_2 = m.$$

1. Find the Marshallian demand functions.
2. Find the indirect utility function.
3. Find $\lambda(\mathbf{p}, m)$. Show that the derivative of the indirect utility function towards m is equal to $\lambda(\mathbf{p}, m)$. Use this to provide an economic interpretation of $\lambda(\mathbf{p}, m)$.
4. Show Roy's identity for $x_1(\mathbf{p}, m)$.

1. Marshallian demand

Lagrangian:

$$\mathcal{L} = x_1^{\alpha} x_2^{1-\alpha} - \lambda\big(p_1 x_1 + p_2 x_2 - m\big).$$

FOCs:

$$\alpha x_1^{\alpha-1} x_2^{1-\alpha} = \lambda p_1, \qquad (1-\alpha) x_1^{\alpha} x_2^{-\alpha} = \lambda p_2, \qquad p_1 x_1 + p_2 x_2 = m.$$

From the first two FOCs:

$$\frac{\alpha}{1-\alpha} \cdot \frac{x_2}{x_1} = \frac{p_1}{p_2} \quad \Rightarrow \quad \frac{x_2}{x_1} = \frac{1-\alpha}{\alpha} \cdot \frac{p_1}{p_2}.$$

Use the budget constraint to solve for x_1 and x_2:

$$x_1(p, m) = \frac{\alpha m}{p_1}, \qquad x_2(p, m) = \frac{(1-\alpha) m}{p_2}.$$

2. Indirect utility function

Plug optimal demands $x^\star$ into the utility function:

$$v(p,m) = u\big(x_1(p,m), x_2(p,m)\big) = \left(\frac{\alpha m}{p_1}\right)^{\alpha} \left(\frac{(1-\alpha)m}{p_2}\right)^{1-\alpha}.$$

2. Indirect utility function

Plug optimal demands $x^\star$ into the utility function:

$$v(p, m) = u\big(x_1(p, m), x_2(p, m)\big) = \left(\frac{\alpha m}{p_1}\right)^{\alpha} \left(\frac{(1-\alpha)m}{p_2}\right)^{1-\alpha}.$$

Simplify, so the indirect utility is

$$v(p, m) = \left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}\, p_1^{-\alpha} p_2^{-(1-\alpha)}\right] m.$$

2. Indirect utility function

Plug optimal demands $x^{\star}$ into the utility function:

$$v(p,m) = u\big(x_1(p,m), x_2(p,m)\big) = \left(\frac{\alpha m}{p_1}\right)^{\alpha}\left(\frac{(1-\alpha)m}{p_2}\right)^{1-\alpha}.$$

Simplify, so the indirect utility is

$$v(p,m) = \left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}\, p_1^{-\alpha} p_2^{-(1-\alpha)}\right] m.$$

In class, we have seen that one can take the log of the utility function. This simplifies a lot the calculations (also for the Marshallian demand): $v(p,m) = \alpha lnm - \alpha lnp_1 - (1-\alpha)lnp_2$. However, the non-transformed version should be used for the next question.

3. Lagrange multiplier and its meaning

We have shown that, following from the envelope theorem, the following holds:

$$\frac{\partial v(p, m)}{\partial m} = \lambda(p, m).$$

We can rewrite the expression for $v(p, m)$ derived in part 2 as:

$$v(p, m) = A(p)\, m, \quad \text{where } A(p) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\, p_1^{-\alpha} p_2^{-(1-\alpha)}.$$

3. Lagrange multiplier and its meaning

We have shown that, following from the envelope theorem, the following holds:

$$\frac{\partial v(p,m)}{\partial m} = \lambda(p,m).$$

We can rewrite the expression for $v(p,m)$ derived in part 2 as:

$$v(p,m) = A(p)\,m, \quad \text{where } A(p) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\,p_1^{-\alpha}p_2^{-(1-\alpha)}.$$

If follows

$$\frac{\partial v(p,m)}{\partial m} = A(p) = \lambda(p,m) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\,p_1^{-\alpha}p_2^{-(1-\alpha)}.$$

3. Lagrange multiplier and its meaning

We have shown that, following from the envelope theorem, the following holds:

$$\frac{\partial v(p,m)}{\partial m} = \lambda(p,m).$$

We can rewrite the expression for $v(p,m)$ derived in part 2 as:

$$v(p,m) = A(p)\,m, \quad \text{where } A(p) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\,p_1^{-\alpha}p_2^{-(1-\alpha)}.$$

If follows

$$\frac{\partial v(p,m)}{\partial m} = A(p) = \lambda(p,m) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\,p_1^{-\alpha}p_2^{-(1-\alpha)}.$$

Economic interpretation

- $\lambda(p,m)$ is the marginal utility of income.
- $\lambda(p,m)$ tells how much utility increases when income m increases by one unit.
- For instance, assume that $\alpha = 1/2$, $p_1 = 1$, $p_2 = 4$, then:

$$\lambda(p) = 0.5^{0.5}(1-0.5)^{1-0.5}\,1^{-0.5}\,4^{-0.5} = 0.25.$$

In this case, utility increases by 0.25 utils when m increases by one unit.

4. Roy's identity for $x_1(p, m)$

Roy's identity:

$$x_1(p, m) = -\frac{\frac{\partial v(p,m)}{\partial p_1}}{\frac{\partial v(p,m)}{\partial m}}.$$

We have just calculated the denominator. Derivative wrt p_1, recall that $v(p, m) = m\alpha^{\alpha}(1-\alpha)^{1-\alpha} p_1^{-\alpha} p_2^{-(1-\alpha)}$, it follows:

4. Roy's identity for $x_1(p, m)$

Roy's identity:

$$x_1(p, m) = -\frac{\frac{\partial v(p, m)}{\partial p_1}}{\frac{\partial v(p, m)}{\partial m}}.$$

We have just calculated the denominator. Derivative wrt p_1, recall that $v(p, m) = m\alpha^{\alpha}(1-\alpha)^{1-\alpha} p_1^{-\alpha} p_2^{-(1-\alpha)}$, it follows:

$$\frac{\partial v}{\partial p_1} = m\alpha^{\alpha}(1-\alpha)^{1-\alpha}(-\alpha)p_1^{-\alpha-1} p_2^{-(1-\alpha)}.$$

4. Roy's identity for $x_1(p, m)$

Roy's identity:

$$x_1(p, m) = -\frac{\frac{\partial v(p,m)}{\partial p_1}}{\frac{\partial v(p,m)}{\partial m}}.$$

We have just calculated the denominator. Derivative wrt p_1, recall that $v(p, m) = m\alpha^\alpha(1-\alpha)^{1-\alpha} p_1^{-\alpha} p_2^{-(1-\alpha)}$, it follows:

$$\frac{\partial v}{\partial p_1} = m\alpha^\alpha(1-\alpha)^{1-\alpha}(-\alpha)p_1^{-\alpha-1} p_2^{-(1-\alpha)}.$$

So

$$\frac{\partial v}{\partial p_1} = -\frac{\alpha m}{p_1}\alpha^\alpha(1-\alpha)^{1-\alpha} p_1^{-\alpha} p_2^{-(1-\alpha)} = -\frac{\alpha m}{p_1} A(p).$$

4. Roy's identity for $x_1(p, m)$

Roy's identity:

$$x_1(p, m) = -\frac{\dfrac{\partial v(p, m)}{\partial p_1}}{\dfrac{\partial v(p, m)}{\partial m}}.$$

We have just calculated the denominator. Derivative wrt p_1, recall that $v(p, m) = m\alpha^{\alpha}(1-\alpha)^{1-\alpha} p_1^{-\alpha} p_2^{-(1-\alpha)}$, it follows:

$$\frac{\partial v}{\partial p_1} = m\alpha^{\alpha}(1-\alpha)^{1-\alpha}(-\alpha)p_1^{-\alpha-1}p_2^{-(1-\alpha)}.$$

So

$$\frac{\partial v}{\partial p_1} = -\frac{\alpha m}{p_1}\alpha^{\alpha}(1-\alpha)^{1-\alpha} p_1^{-\alpha} p_2^{-(1-\alpha)} = -\frac{\alpha m}{p_1}A(p).$$

Apply Roy's identity:

$$x_1(p, m) = -\frac{\partial v/\partial p_1}{\partial v/\partial m} = -\frac{-\frac{\alpha m}{p_1}A(p)}{A(p)} = \frac{\alpha m}{p_1},$$

which is the Marshallian demand found in part 1.

Fifth question: Chapter 7, slide 48.

Expenditure minimization (1 and 2)

$$\min_{x_1,x_2} p_1x_1 + p_2x_2 \quad \text{s.t.} \quad x_1^{\alpha}x_2^{1-\alpha} = u$$

Lagrangian:

$$\mathcal{L} = p_1x_1 + p_2x_2 - \lambda\left(x_1^{\alpha}x_2^{1-\alpha} - u\right).$$

FOCs:

$$p_1 = \lambda\alpha x_1^{\alpha-1}x_2^{1-\alpha}, \qquad p_2 = \lambda(1-\alpha)x_1^{\alpha}x_2^{-\alpha}, \qquad x_1^{\alpha}x_2^{1-\alpha} = u$$

Taking ratios:

$$\frac{p_1}{p_2} = \frac{\alpha}{1-\alpha}\frac{x_2}{x_1} \quad \Rightarrow \quad x_2 = \frac{1-\alpha}{\alpha}\frac{p_1}{p_2}x_1.$$

Substitute into the constraint:

$$x_1^{\alpha}\left(\frac{1-\alpha}{\alpha}\frac{p_1}{p_2}x_1\right)^{1-\alpha} = u \quad \Longleftrightarrow \quad h_1(p,u) = u\left(\frac{\alpha}{1-\alpha}\frac{p_2}{p_1}\right)^{1-\alpha},$$

$$h_2(p,u) = u\left(\frac{1-\alpha}{\alpha}\frac{p_1}{p_2}\right)^{\alpha}.$$

Expenditure function:

$$e(p,u) = p_1x_1^h(p,u) + p_2x_2^h(p,u) = u\,\frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}}p_1^{\alpha}p_2^{1-\alpha}.$$